Invariant Families Across Collapse Classes in Quantum Collapse Geometry

Stephen Garner

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Abstract

Quantum Collapse Geometry (QCG) has previously been developed through specific collapse-selection operators acting on relational configurations. While these constructions reproduce measurement-like outcomes, interference structure, and statistical behavior in minimal systems, they risk overcommitting to particular dynamical forms. In this paper, we reformulate QCG at a higher level of abstraction by introducing collapse classes—families of admissible collapse dynamics defined by structural constraints—and identifying invariant families preserved or selected across these classes.

We show that key physical structures—measurement sectors, topological invariants, admissible attractors, and Born-like weighting—can be understood as emergent from constraints on these invariant families rather than from any specific collapse operator. This reframing aligns QCG with its underlying ontology, in which collapse is a universal update grammar and physical law emerges from stable relational structure under finite invariance.

This formulation provides a unified framework for interpreting previous results and establishes a program for deriving effective physical laws from structural invariance across collapse dynamics.

1 Introduction: From Operators to Structure

Previous work in Quantum Collapse Geometry (QCG) has focused on the construction and analysis of specific collapse operators acting on relational configurations. These constructions have demonstrated that collapse-selection dynamics can reproduce key features associated with physical systems, including measurement-like outcomes, interference structure, and emergent statistical behavior in minimal models.

However, this operator-centric approach introduces a limitation. By emphasizing particular functional forms of collapse dynamics, it risks overfitting the theory to specific update rules whose physical status remains unclear. In particular, it raises the question of whether any single collapse operator should be regarded as fundamental, or whether the relevant physical content lies at a more abstract level.

The ontology underlying QCG suggests the latter. In this framework, collapse is not a specialized mechanism associated with measurement, but a universal update grammar acting on relational structure. Physical behavior arises not from the detailed form of a single operator, but from the persistence of relational structure under admissible transformations.

This observation motivates a shift in perspective. Rather than asking which collapse operator governs physical systems, we ask:

> Which relational structures are preserved or selected across admissible collapse dynamics?

The aim of this work is to formalize this shift. We introduce the notion of *collapse classes*, defined by structural conditions shared across admissible collapse dynamics, and identify *invariant families*—collections of relational structures that persist across these classes.

Within this formulation, familiar physical features can be reinterpreted as manifestations of invariant structure:

- measurement arises from fixed-point sector formation under collapse,
- probability emerges as an invariant weighting over attractor basins,
- topological quantities are preserved as global invariant families,
- energy and force correspond to local relational tension and its gradients.

These results suggest a reframing of physical theory. Rather than identifying physical law with a specific dynamical equation, we propose that:

> Physical law corresponds to invariant relational structure preserved across admissible collapse dynamics under finite invariance.

This perspective aligns QCG with structural approaches in modern physics, in which symmetry, invariance, and effective description play central roles. It also provides a framework for organizing previous results, which are here reinterpreted as realizations of invariant families within particular subclasses of collapse dynamics.

The structure of the paper is as follows. Section 2 briefly recalls the ontological foundations of QCG. Section 3 defines the relational configuration space and its projection to observable description. Section 4 introduces collapse classes, and Section 5 formalizes invariant families. Section 6 identifies representative invariant structures, including local alignment, attractor formation, measurement sectors, topological invariants, and Born-like weighting. Section 7 presents explicit collapse operators as realizations of these structures. Section 8 discusses the emergence of physical quantities, and Sections 9–10 address interpretation, limitations, and future directions.

The goal of this work is not to replace existing physical theories, but to provide a structural framework within which their key features can be understood as arising from invariant relational structure under admissible collapse dynamics.

2 Ontological Grounding

The formulation developed in this work builds on the ontological framework introduced in earlier QCG papers. We briefly summarize the key principles required for the present analysis, emphasizing their structural role.

2.1 Collapse as Universal Update Grammar

In QCG, collapse is not a process specific to measurement, but a universal mechanism by which relational configurations are updated.

Collapse acts to prune, stabilize, or reorganize relational structure, selecting configurations that are consistent under admissible constraints. It is therefore best understood as an update grammar governing the evolution of relational configurations, rather than as a dynamical event tied to observation.

This interpretation removes the need to treat measurement as a special operation, and instead places all physical evolution within a single class of collapse-selection processes.

2.2 Phase as Relational Compatibility

Phase variables in QCG do not represent oscillatory quantities in a fundamental sense. Rather, they encode relational compatibility between degrees of freedom.

Two configurations are considered compatible to the extent that their relational phases permit coherent coexistence under collapse dynamics. Phase differences therefore measure the degree of relational distinguishability or tension between configurations.

In this interpretation, phase is a structural property governing which configurations can persist together, rather than a primitive dynamical variable.

2.3 Finite Invariance

A central principle of QCG is that relational structure is subject to finite invariance: not all distinctions can be resolved arbitrarily.

Under collapse dynamics, configurations that differ only below a certain admissible resolution are treated as equivalent. This induces an effective equivalence relation on Σ and defines the admissible set of configurations.

Finite invariance has two key consequences:

- it prevents the indefinite refinement of relational distinctions,
- it introduces multiplicity within admissible configurations, giving rise to ensemble structure.

Thus, admissibility is not imposed externally, but emerges from the limits of distinguishability under collapse.

2.4 Emergence Across Scale

Structures commonly treated as fundamental in physical theories arise in QCG as emergent features of relational coherence across scale.

- **Time** emerges from the ordered propagation of coherent collapse updates,
- **Geometry** emerges from stabilized coupling relations between degrees of freedom,
- **Force** emerges as the gradient of relational tension,
- **Energy** emerges as a measure of local inconsistency.

These quantities are not introduced as primitives, but as descriptors of invariant relational structure that persists under admissible collapse dynamics.

2.5 Role in the Present Work

The principles above define the conceptual foundation for the collapse-class formulation developed in subsequent sections.

In particular:

- collapse as a universal update grammar motivates the introduction of collapse classes,
- phase as relational compatibility underlies the definition of discrepancy and coherence,

- finite invariance provides the basis for admissibility and ensemble structure,
- emergence across scale justifies the identification of physical quantities with invariant families.

The remainder of this work builds on these principles to formalize invariant relational structure across admissible collapse dynamics.

3 Relational State Space and Descriptive Projection

To formalize the collapse-class framework, we introduce a general relational configuration space together with a projection to observable description. This separation between underlying relational structure and effective observables is central to the formulation of invariant families.

3.1 Relational Configuration Space Σ

Let Σ denote a relational configuration space. Elements $x \in \Sigma$ represent configurations of relational degrees of freedom, without commitment to a specific parametrization or representation.

Depending on the context, Σ may take the form of:

- discrete lattices with phase-like variables,
- graphs with weighted or directional relational structure,
- continuous fields defined over a spatial domain,
- more general relational data encoding compatibility constraints.

The defining feature of Σ is that its elements represent *relations* rather than intrinsic properties of isolated components. Physical structure is therefore encoded in the configuration of relations across the system.

3.2 Descriptive Projection $P : \Sigma \to \mathcal{O}$

Observable descriptions arise through a projection

$$P : \Sigma \to \mathcal{O},$$

where $\mathcal{O}$ denotes a space of effective or observable states.

This projection is generally many-to-one: distinct relational configurations may be indistinguishable at the level of observable description. The projection P therefore implements a form of coarse-graining, mapping fine-grained relational structure to effective physical observables.

In this framework:

- Σ encodes the full relational configuration,
- $\mathcal{O}$ encodes the accessible descriptive structure,
- P mediates between these levels.

Physical measurements correspond to statements about $P(x)$ rather than about x itself.

3.3 Admissibility

A central concept in QCG is that not all configurations in Σ are physically realizable at the level of observable structure. Instead, admissibility is determined by stability under collapse dynamics.

Let $\mathcal{C}$ be a collapse class as defined in Section 4. For $\Phi \in \mathcal{C}$, define the admissible set

$$A_\Phi = \{x \in \Sigma \mid \lim_{n\to\infty} \Phi^n(x) \in \mathrm{Inv}(\Phi)\}.$$

More generally, we write

$$A = \bigcup_{\Phi \in \mathcal{C}'} A_\Phi,$$

for an admissible subclass $\mathcal{C}' \subseteq \mathcal{C}$.

Interpretation. The admissible set consists of configurations that are stable under collapse dynamics, either as fixed points or as members of invariant attractor families.

These configurations may be interpreted as:

- collapse-stable sectors of relational configuration space,
- physically realizable configurations under admissible dynamics,
- equivalence classes under finite invariance.

Admissible Equivalence. Finite invariance induces an equivalence relation on Σ, under which configurations that converge to the same invariant sector are identified:

$$x \sim_A y \quad \Longleftrightarrow \quad \lim_{n\to\infty} \Phi^n(x) = \lim_{n\to\infty} \Phi^n(y)$$

up to admissible resolution.

This equivalence relation defines the effective structure of the admissible set.

Relation to Observables. The projection P is assumed to respect admissible equivalence in the sense that

$$x \sim_A y \;\Rightarrow\; P(x) \approx P(y),$$

up to the resolution of $\mathcal{O}$.

Thus, observable states correspond not to individual configurations in Σ, but to equivalence classes of admissible configurations under collapse dynamics.

3.4 Role in the Present Framework

The structures introduced here provide the foundation for subsequent sections:

- collapse classes (Section 4) act on Σ,
- invariant families (Section 5) are defined over admissible structure in Σ,
- observable quantities arise from projection P applied to invariant sectors,
- probability and measurement emerge from the partition of Σ into admissible equivalence classes.

This separation between relational configuration space and descriptive projection allows physical structure to be understood as emerging from invariant relational properties rather than from a fixed underlying representation.

4 Collapse Classes

Previous formulations of Quantum Collapse Geometry (QCG) have introduced specific collapse operators acting on relational configurations. While such constructions are useful for illustrating particular phenomena, they risk overcommitting to a single dynamical form. The ontology developed in earlier work does not privilege any specific operator, but instead treats collapse as a universal update mechanism acting on relational structure.

To reflect this, we introduce the notion of *collapse classes*: families of admissible collapse dynamics defined by shared structural properties rather than by explicit functional form.

4.1 Relational Configuration Space

Let Σ denote a relational configuration space. Elements $x \in \Sigma$ represent configurations of relational degrees of freedom, which may include discrete phase variables, continuous fields, coupling structures, or other relational data. No fixed parametrization is assumed; only relational structure is taken as fundamental.

Observable descriptions arise through a projection

$$P : \Sigma \to \mathcal{O},$$

where $\mathcal{O}$ is a space of coarse-grained or effective observables. This projection is generally many-to-one and reflects finite resolution under admissible description.

4.2 Definition of Collapse Classes

A *collapse class* $\mathcal{C}$ is a collection of maps

$$\Phi : \Sigma \to \Sigma$$

satisfying a common set of structural conditions. Each $\Phi \in \mathcal{C}$ represents a possible realization of collapse-selection dynamics.

The role of $\mathcal{C}$ is not to identify a unique fundamental law, but to define a family of admissible dynamics within which invariant relational structure can be studied.

4.3 Structural Conditions

We impose the following minimal conditions on $\mathcal{C}$:

1. **Relational Locality.** For each $\Phi \in \mathcal{C}$ and each degree of freedom indexed by i, the update $\Phi(x)_i$ depends only on a localized neighborhood of relational structure around i, or on a kernel with finite support in the continuum limit.

2. **Finite Invariance Compatibility.** Collapse dynamics respect finite resolution: they do not generate arbitrarily fine distinctions in relational structure. Configurations indistinguishable under admissible resolution remain so under iteration of Φ.

3. **Attractor Formation.** For each $\Phi \in \mathcal{C}$, repeated iteration partitions Σ into invariant sectors (fixed points or attractor families). That is, for generic $x \in \Sigma$, the sequence $\Phi^n(x)$ converges to a subset $\mathrm{Inv}(\Phi) \subseteq \Sigma$.

4. **Non-Expansive Relational Tension.** There exists a class of discrepancy or tension functionals $D : \Sigma \to \mathbb{R}_{\geq 0}$ such that, for $\Phi \in \mathcal{C}$, iteration does not generically increase relational inconsistency. In representative cases, D is reduced under application of Φ.

5. **Coarse-Graining Compatibility.** Collapse dynamics are consistent with projection in the sense that, for appropriate regimes,

$$P \circ \Phi \approx \Phi_{\mathrm{eff}} \circ P,$$

   where Φ_{eff} is an induced effective map on $\mathcal{O}$. This ensures that stable structure persists under descriptive coarse-graining.

These conditions are intentionally minimal. They define a broad class of admissible collapse dynamics while leaving room for substantial variation in explicit form.

4.4 Admissible Set and Invariant Sectors

For each $\Phi \in \mathcal{C}$, define the invariant set

$$\mathrm{Inv}(\Phi) = \{x \in \Sigma \mid \Phi(x) = x\},$$

together with more general attractor families obtained as limits of iteration.

We define the *admissible set* associated with Φ as

$$A_\Phi = \{x \in \Sigma \mid \lim_{n\to\infty} \Phi^n(x) \in \mathrm{Inv}(\Phi)\}.$$

Finite invariance implies that A_Φ typically contains multiplicity: multiple distinct configurations may map to the same invariant sector under admissible resolution. This multiplicity plays a central role in the emergence of ensemble structure.

4.5 Subclasses of Collapse Dynamics

Within $\mathcal{C}$, it is useful to distinguish subclasses characterized by additional structure:

- $\mathcal{C}_{\mathrm{disc}}$: collapse maps defined on discrete lattices or graphs, with strictly local neighborhood dependence.

- $\mathcal{C}_{\text{cont}}$: collapse maps defined on continuous configuration spaces or fields, typically expressed through integral kernels with localized support.
- $\mathcal{C}_{\text{top}}$: maps that preserve topological invariants of the configuration (e.g., winding number or homotopy class).
- $\mathcal{C}_{\text{meas}}$: maps that produce well-defined fixed-point sectors corresponding to measurement-like outcomes.

These subclasses are not disjoint and may overlap. Their purpose is to organize the analysis of invariant families across different dynamical regimes.

4.6 Role of Collapse Classes

The introduction of collapse classes shifts the focus of QCG from specific dynamical rules to structural properties shared across admissible dynamics. Rather than asking which collapse operator is fundamental, we ask:

> Which relational structures are preserved or selected across collapse classes satisfying the conditions above?

This perspective allows physical law to be formulated in terms of invariant families across $\mathcal{C}$, rather than in terms of a privileged update rule. The subsequent sections develop this program by identifying such invariant structures and relating them to measurement, probability, topology, and emergent physical description.

5 Invariant Families

The introduction of collapse classes shifts the central question of Quantum Collapse Geometry (QCG) from the specification of a particular dynamical rule to the identification of *structures that persist across admissible collapse dynamics*. These persistent structures are formalized here as *invariant families*.

5.1 Definition

Let Σ be a relational configuration space and $\mathcal{C}$ a collapse class as defined in Section 4. Let $\mathcal{C}' \subseteq \mathcal{C}$ denote a subclass of collapse maps sharing additional structural properties.

An *invariant family* $\mathcal{I}$ over Σ is a collection of relational structures (e.g., subsets, equivalence classes, functionals, or measures) such that for all $\Phi \in \mathcal{C}'$,

$$x \in \mathcal{I} \;\Rightarrow\; \Phi(x) \in \mathcal{I},$$

or, more generally, such that $\mathcal{I}$ is preserved under iteration of Φ up to admissible equivalence.

In this sense, invariant families capture those aspects of relational configuration that remain stable under collapse dynamics across a class of admissible update rules.

5.2 Admissible Equivalence

Due to finite invariance, exact equality of configurations is neither required nor generally meaningful at all scales. Instead, invariance is defined relative to an admissible equivalence relation $\sim_A$ on Σ, under which configurations indistinguishable at the relevant resolution are identified.

An invariant family $\mathcal{I}$ is therefore understood as preserved if

$$x \in \mathcal{I} \;\Rightarrow\; \Phi(x) \sim_A y \quad \text{for some } y \in \mathcal{I}.$$

This formulation allows invariant structure to persist under coarse-graining and finite resolution, rather than requiring exact pointwise invariance.

5.3 Taxonomy of Invariant Families

Invariant families in QCG arise at multiple structural levels. We distinguish the following categories:

(A) Local Invariants. These are functionals or constraints defined on local relational neighborhoods that are preserved or systematically reduced under collapse dynamics. Examples include discrepancy or tension measures that characterize local relational inconsistency.

(B) Global Invariants. These are structures defined over extended regions of Σ, such as topological invariants (e.g., winding number, homotopy class) that remain stable under admissible collapse maps, particularly those in subclasses preserving global relational constraints.

(C) Attractor Invariants. Collapse dynamics partition Σ into basins of attraction associated with invariant sectors. The resulting attractor structure—including fixed points, limit sets, and basin boundaries—forms an invariant family under $\mathcal{C}'$.

(D) Measure Invariants. Given the multiplicity of admissible configurations under finite invariance, ensemble structure arises over Σ. Measures defined over attractor basins or admissible sets that remain consistent under collapse-class transformations form invariant families.

(E) Descriptive Invariants. Under projection $P : \Sigma \to \mathcal{O}$, invariant families induce stable observable structures. These include measurement sectors, effective states, and coarse-grained descriptions that are robust under admissible collapse dynamics.

5.4 Invariant Families and Emergence

Invariant families provide the mechanism by which higher-level physical structure emerges from collapse dynamics.

Rather than being imposed as fundamental primitives, quantities such as energy, charge, measurement outcomes, and probability distributions correspond to invariant relational structures that persist across collapse classes.

In this sense, emergence in QCG is not attributed to approximation or ignorance, but to the existence of stable invariant families under admissible dynamics. A structure is physically meaningful if and only if it is preserved, or systematically selected, across a sufficiently broad subclass of $\mathcal{C}$.

5.5 Stability Across Collapse Classes

A central requirement is that physically meaningful invariant families be *robust* with respect to variation within the collapse class.

Let $\mathcal{I}$ be an invariant family associated with a subclass $\mathcal{C}' \subseteq \mathcal{C}$. We say that $\mathcal{I}$ is *collapse-class stable* if it is preserved under all $\Phi \in \mathcal{C}'$ satisfying the structural conditions of Section 4.

This notion of stability replaces the requirement of exact conservation under a single dynamical law with invariance across a class of admissible dynamics.

5.6 Relation to Subsequent Sections

The remainder of this work identifies specific invariant families corresponding to familiar physical structures:

- Local alignment and discrepancy reduction (Section 6.1),
- Attractor structure and admissibility (Section 6.2),
- Measurement sectors as fixed-point families (Section 6.3),
- Topological invariants under collapse (Section 6.4),
- Born-like weighting as a measure invariant (Section 6.5).

Each of these results should be understood not as a consequence of a particular collapse operator, but as an instance of invariant structure arising across admissible collapse classes.

5.7 Interpretation

The introduction of invariant families allows QCG to be formulated at the appropriate level of abstraction. Collapse dynamics define a space of possible evolutions, but the physically meaningful content of the theory resides in the structures that remain stable across that space.

In this formulation, physical law is not identified with a specific dynamical equation, but with the invariant relational structures selected by admissible collapse dynamics under finite invariance.

6 Representative Invariant Families Across Collapse Classes

We now identify specific invariant families that arise across subclasses of admissible collapse dynamics. These results illustrate how familiar structural features emerge not from a particular collapse operator, but from properties shared across collapse classes.

6.1 Local Alignment and Discrepancy Reduction

A broad subclass of collapse maps exhibits a tendency toward local relational alignment. This behavior can be characterized in terms of discrepancy or tension functionals defined over Σ.

Discrepancy Functionals. Let $D : \Sigma \to \mathbb{R}_{\geq 0}$ be a functional measuring local relational inconsistency. In discrete phase systems, a representative example is

$$D[x] = \sum_{\langle i,j \rangle \in E} \left(1 - \cos(\theta_i - \theta_j)\right),$$

which vanishes when neighboring phases are aligned.

Local Alignment Property. Consider collapse maps $\Phi \in \mathcal{C}_{\text{disc}}$ defined by local aggregation of neighboring configurations. For a wide class of such maps, each update step selects a local configuration that maximizes alignment with its neighborhood.

More precisely, for each site i, define a local alignment functional

$$A_i(\phi) = \sum_{j \sim i} \cos(\phi - \theta_j).$$

Then the update rule

$$\Phi(x)_i = \arg\left(\sum_{j \sim i} e^{i\theta_j}\right)$$

maximizes $A_i(\phi)$ with respect to $\phi \in S^1$.

This property is not unique to a single operator, but holds for a class of collapse maps that implement local consistency selection.

Invariant Family. The class of discrepancy functionals D defines a local invariant family in the following sense:

- D is non-negative and vanishes on aligned configurations,
- collapse maps in the relevant subclass do not generically increase D,
- iteration tends to reduce D or confine dynamics to low-discrepancy regions.

While strict monotonicity of D under all $\Phi \in \mathcal{C}$ is not assumed, numerical and analytical results in representative cases show consistent relaxation toward low-discrepancy configurations.

Interpretation. Local alignment and discrepancy reduction represent a structural invariant across collapse classes: regardless of the specific form of Φ, admissible collapse dynamics tend to suppress local relational inconsistency and favor configurations of higher coherence.

This behavior underlies the emergence of stable relational structures and provides the basis for interpreting collapse as a relaxation process in relational space.

6.2 Attractor Structure and Admissibility

A second invariant family arises from the global organization of configuration space under iteration of collapse dynamics.

Attractor Structure. For $\Phi \in \mathcal{C}$ satisfying the conditions of Section 4, repeated application partitions Σ into basins of attraction:

$$\Sigma = \bigsqcup_k B_k,$$

where each basin B_k consists of configurations that converge to a corresponding invariant sector (fixed point or attractor family) under iteration of Φ.

The collection of basins $\{B_k\}$ and their associated invariant sectors form an attractor structure on Σ.

Admissible Set. Define the admissible set associated with Φ as

$$A_\Phi = \{x \in \Sigma \mid \lim_{n\to\infty} \Phi^n(x) \in \mathrm{Inv}(\Phi)\}.$$

Finite invariance implies that collapse does not fully resolve all distinctions. As a result, distinct configurations may converge to the same invariant sector under admissible resolution.

This induces an equivalence relation on Σ:

$$x \sim_A y \quad \iff \quad \lim_{n\to\infty} \Phi^n(x) = \lim_{n\to\infty} \Phi^n(y)$$

up to admissible equivalence.

Invariant Family. The attractor structure defines an invariant family consisting of:

- invariant sectors $\mathrm{Inv}(\Phi)$,
- basins of attraction $\{B_k\}$,
- equivalence classes under $\sim_A$.

This structure is preserved across subclasses of collapse maps that share attractor formation and finite invariance properties.

Stability Across Collapse Classes. While the detailed geometry of basins may vary with Φ, the existence of a partition of Σ into attractor sectors is robust across $\mathcal{C}' \subseteq \mathcal{C}$. In this sense, admissibility is not tied to a particular operator, but to the general existence of attractor structure under collapse dynamics.

Interpretation. Admissibility can therefore be identified with attractor structure: configurations are physically meaningful to the extent that they belong to stable sectors under collapse.

This identification provides the foundation for subsequent constructions: measurement sectors correspond to attractor families, and ensemble structure arises from multiplicity within admissible equivalence classes.

Connection to Subsequent Results. The attractor-based notion of admissibility plays a central role in the emergence of probability. In particular, the partition of Σ into basins $\{B_k\}$ induces a natural framework for defining outcome frequencies, which will be developed in Section 6.5.

6.3 Measurement as Fixed-Point Sector Formation

Within the collapse-class formulation, measurement is not introduced as a primitive operation or an external intervention. Instead, it arises as a consequence of the attractor structure of collapse dynamics under admissible projection.

Fixed-Point Sectors. Let $\Phi \in \mathcal{C}$ and consider its invariant set

$$\mathrm{Inv}(\Phi) = \{x \in \Sigma \mid \Phi(x) = x\},$$

together with more general attractor families obtained as limits of iteration.

These invariant configurations define *fixed-point sectors* of the relational configuration space. Each sector corresponds to a class of configurations that are stable under collapse dynamics up to admissible equivalence.

Sector Formation. As shown in Section 6.2, iteration of Φ partitions Σ into basins of attraction

$$\Sigma = \bigsqcup_k B_k,$$

where each basin B_k converges to a corresponding invariant sector.

This partition is robust across subclasses of collapse maps satisfying attractor formation and finite invariance. In this sense, sector formation is an invariant structural feature of admissible collapse dynamics.

Projection to Observables. Let $P : \Sigma \to \mathcal{O}$ be the coarse-graining map to observable descriptions. We assume that P is approximately constant on each basin B_k up to admissible resolution, so that

$$x \in B_k \;\Rightarrow\; P(x) \approx o_k,$$

for some observable label $o_k \in \mathcal{O}$.

Under this condition, each attractor sector induces a well-defined observable outcome. The mapping

$$B_k \;\mapsto\; o_k$$

defines the observable structure associated with collapse dynamics.

Measurement as Sector Selection. Measurement can therefore be identified with the process by which a configuration $x \in \Sigma$ is mapped, under iteration of Φ, into a basin B_k and subsequently into an observable label o_k under projection.

That is, measurement corresponds to:

- convergence of x to an invariant sector under collapse dynamics,
- identification of that sector with an observable outcome via projection.

No additional postulate of state reduction is required; collapse dynamics themselves implement the selection of a stable sector.

Invariant Family. The collection of fixed-point sectors $\{\mathrm{Inv}_k(\Phi)\}$, together with their associated basins $\{B_k\}$ and observable labels $\{o_k\}$, forms an invariant family under subclasses of $\mathcal{C}$ satisfying:

- attractor formation,
- finite invariance,
- coarse-graining compatibility.

While the detailed geometry of basins and the specific form of Φ may vary, the existence of sector formation and its interpretation as measurement is robust across this class.

Determinism and Apparent Randomness. At the level of Σ, the mapping $x \mapsto B_k$ is deterministic for fixed Φ. However, due to finite invariance and unresolved multiplicity in admissible configurations, an effective ensemble description arises over Σ.

As a result, measurement outcomes appear probabilistic when described at the level of observable structure. This probabilistic behavior reflects the distribution of configurations across basins of attraction rather than intrinsic stochasticity in the underlying dynamics.

Relation to Quantum Measurement. In standard quantum mechanics, measurement is represented by projection onto eigenstates of an observable. In the present framework, this structure is reinterpreted as follows:

- eigenstates correspond to invariant sectors of collapse dynamics,
- projection corresponds to coarse-grained identification of attractor basins,

- outcome probabilities arise from the measure of configurations within each basin.

This correspondence does not constitute a derivation of quantum measurement, but provides a structural mapping between collapse-class invariants and the standard formalism.

Connection to Probability. The partition of Σ into basins $\{B_k\}$ provides the natural framework for defining outcome frequencies:

$$f_k = \int_{B_k} \rho(x)\, d\Sigma,$$

where ρ is an admissible measure over configurations.

The structure of this measure and its constraints will be examined in Section 6.5, where Born-like scaling is identified as an invariant weighting family across collapse classes.

Interpretation. Measurement is thus not a fundamental operation imposed on the system, but an emergent feature of collapse dynamics:

> Measurement corresponds to the identification of invariant relational structure under admissible projection.

In this formulation, the apparent discontinuity of measurement reflects the transition from fine-grained relational description to coarse-grained invariant sectors, rather than a special dynamical process distinct from collapse itself.

6.4 Topological Invariants Under Collapse Dynamics

A central requirement for any collapse-based framework is that it be capable of preserving nontrivial global structure while suppressing local inconsistency. Invariant families of topological type provide a clear demonstration that collapse dynamics need not be purely dissipative, but can instead preserve robust global features of relational configurations.

Topological Structure in Relational Configurations. Let Σ be a relational configuration space equipped with phase-like variables defined over a discrete or continuous domain. In many such systems, global structure can be characterized by topological invariants, such as winding number, homotopy class, or more general cohomological data.

For example, in a discrete phase system defined on a closed loop γ, one may define a winding number

$$Q_\gamma = \frac{1}{2\pi} \sum_{(i,j)\in\gamma} \arg\left(e^{i(\theta_j - \theta_i)}\right),$$

which takes values in $\mathbb{Z}$ and is invariant under continuous deformations of the phase configuration.

Compatibility with Collapse Dynamics. Consider collapse maps $\Phi \in \mathcal{C}_{\mathrm{top}} \subseteq \mathcal{C}$ that satisfy:

- relational locality,
- finite invariance,
- non-expansive local discrepancy,
- continuity of updates with respect to admissible equivalence.

Under these conditions, local updates act to reduce phase discrepancies without introducing discontinuities that would change global topological structure.

Invariant Family. Topological quantities such as Q_γ define an invariant family in the following sense:

- local collapse updates smooth phase variation while preserving global winding,
- configurations remain within the same topological sector under iteration of Φ,
- attractor structure (Section 6.2) is refined within each topological class rather than across classes.

Thus, the configuration space Σ is partitioned not only into attractor basins, but also into topological sectors that are preserved across admissible collapse dynamics.

Local Smoothing vs. Global Preservation. This coexistence of local smoothing and global preservation is a key structural feature. Collapse dynamics reduce local relational inconsistency (Section 6.1) while leaving invariant global topological constraints.

In this sense, collapse acts as a *structure-selective* process: it eliminates unstable local variation without erasing globally consistent relational organization.

Interaction with Attractor Structure. Topological sectors constrain attractor formation. That is, for $\Phi \in \mathcal{C}_{\text{top}}$, each basin of attraction B_k lies entirely within a fixed topological class.

Consequently, the admissible set decomposes as

$$A_\Phi = \bigsqcup_{\alpha} A_\Phi^{(\alpha)},$$

where each $A_\Phi^{(\alpha)}$ corresponds to a distinct topological sector labeled by invariant data (e.g., winding number).

This decomposition induces a refinement of the equivalence relation $\sim_A$ introduced in Section 5, incorporating both attractor convergence and topological classification.

Continuum Limit. In continuous systems, analogous results hold for phase fields $\theta(x)$ defined over a domain $\Omega \subset \mathbb{R}^n$. Topological invariants correspond to elements of homotopy or cohomology groups (e.g., $\pi_1(S^1)$ for phase fields), and are preserved under collapse maps that act as local smoothing flows without singularity formation.

In this regime, collapse dynamics resemble diffusion-like processes (cf. Section 4), yet retain global topological constraints, indicating that smoothing alone does not determine the full structure of the system.

Relation to Physical Quantities. In the QCG framework, such topological invariants correspond to physically meaningful quantities. For example, phase winding can be interpreted as a form of conserved charge, with quantization arising naturally from the discrete nature of topological classes.

This provides a structural origin for quantization that does not rely on imposed algebraic conditions, but instead follows from the preservation of invariant families under collapse.

Interpretation. The existence of topological invariant families demonstrates that collapse dynamics are not purely dissipative. Instead, they selectively preserve relational structure that is globally consistent and robust under local updates.

> Collapse eliminates local inconsistency while preserving globally coherent relational structure.

This property is essential for the emergence of stable, particle-like excitations and long-lived physical structures within the QCG framework.

Scope and Limitations. The analysis presented here is schematic and does not constitute a general proof of topological invariance for all collapse maps. In particular:

- invariance depends on continuity properties of Φ,
- singular updates or discontinuities may alter topological class,
- extension to higher-dimensional and non-abelian structures requires further work.

Nevertheless, representative models demonstrate that topological invariants form a robust family of structures preserved across admissible subclasses of collapse dynamics.

6.5 Born-like Scaling as an Invariant Family

A central open question for any collapse-based framework is the origin of probability weighting. In earlier constructions, outcome frequencies were represented as measures over basins of attraction under repeated application of collapse dynamics. However, the specific form of that measure was left undetermined. Within the present formulation, this question can be sharpened.

Rather than asking which probability rule is generated by a particular collapse operator, we ask:

> Which weighting functions on invariant sectors are preserved across admissible collapse classes under structural constraints of symmetry, coarse-graining, and finite invariance?

We show that, for two-sector systems, these constraints select a unique linear weighting family. When the sector weights are identified with squared amplitudes in the standard quantum description, this yields Born-like scaling.

Invariant Sectors and Basin Weights. Let $\Phi \in \mathcal{C}$ and suppose that iteration of Φ partitions the relational configuration space Σ into basins of attraction

$$\Sigma = \bigsqcup_k B_k,$$

each associated with an invariant sector $\mathrm{Inv}_k(\Phi)$ as described in Sections 6.2 and 6.3.

Let $w_k \geq 0$ denote the structural weight associated with sector k, with

$$\sum_k w_k = 1.$$

In minimal two-sector systems we write

$$(w_0, w_1), \qquad w_0, w_1 \geq 0, \qquad w_0 + w_1 = 1.$$

Let μ be a weighting functional assigning effective outcome weights to sectors, so that the corresponding frequencies are

$$f_k = \mu(w_k).$$

More generally, for a set of sectors $\{k\}$,

$$f_k = \int_{B_k} \rho(x)\, d\Sigma,$$

where ρ is an admissible measure over unresolved configurations. The role of μ is to characterize the induced weighting at the level of invariant sectors themselves.

Structural Constraints. We impose the following conditions on μ.

1. **Normalization.** For a complete set of sectors,

$$\sum_k \mu(w_k) = 1.$$

2. **Label Symmetry.** If two sectors are exchanged by relabeling, their weights are exchanged accordingly. In particular, for two-sector systems,

$$\mu(w_0) = \mu(w_1) \quad \text{whenever} \quad w_0 = w_1.$$

3. **Coarse-Graining Additivity.** If a collection of sectors is grouped into a single effective sector, the weight of the aggregate equals the sum of the weights of its constituents. Thus, for disjoint sectors with structural weights u and v,

$$\mu(u + v) = \mu(u) + \mu(v),$$

whenever $u + v \leq 1$.

4. **Parameterization Invariance.** The weighting depends only on relational sector weight, not on the particular parameterization of Σ used to describe the unresolved configurations giving rise to that weight.

5. **Collapse-Class Robustness.** The same functional form of μ must be preserved across all $\Phi \in \mathcal{C}'$, for some admissible subclass $\mathcal{C}' \subseteq \mathcal{C}$ satisfying the structural conditions of

Section 4.

Proposition. Under the constraints above, the weighting functional μ is linear:

$$\mu(w) = w$$

for all sector weights $w \in [0, 1]$.

Proof. We proceed in stages.

(i) Vanishing at zero and unit normalization. By normalization, a null sector contributes no weight, so

$$\mu(0) = 0.$$

Likewise, a single complete sector must have total weight one, so

$$\mu(1) = 1.$$

(ii) Additivity. By coarse-graining consistency, whenever $u, v \geq 0$ and $u + v \leq 1$,

$$\mu(u + v) = \mu(u) + \mu(v).$$

Thus μ is additive on $[0, 1]$.

(iii) Rational linearity. For any positive integer n, additivity gives

$$\mu\left(\frac{1}{n}\right) = \frac{1}{n}\mu(1) = \frac{1}{n}.$$

For any rational number $m/n \in [0, 1]$,

$$\mu\left(\frac{m}{n}\right) = m\,\mu\left(\frac{1}{n}\right) = \frac{m}{n}.$$

(iv) Extension to all admissible weights. Parameterization invariance and collapse-class robustness rule out pathological dependence on special coordinate choices or collapse-map details. Since sector weights represent coarse-grained relational proportions, admissible weighting must vary consistently under refinement of the partition. Accordingly, the rational result extends uniquely to all $w \in [0, 1]$:

$$\mu(w) = w.$$

This proves linearity. $\square$

Born-like Scaling. For a two-sector system, it follows immediately that

$$f_0 = w_0, \qquad f_1 = w_1.$$

If the sector weights are identified with squared amplitudes in the standard quantum representation,

$$w_k = |\alpha_k|^2,$$

then

$$f_k = |\alpha_k|^2,$$

which matches the Born rule.

Interpretation. The significance of this result is not that the Born rule is postulated, but that Born-like scaling appears as the unique invariant weighting family compatible with:

- the existence of invariant sectors under collapse dynamics,
- additivity under admissible coarse-graining,
- symmetry between equivalent sectors,
- and robustness across admissible collapse classes.

Probability is therefore not introduced as intrinsic randomness at the level of fundamental dynamics. Rather, it arises as a structural weighting on invariant sectors induced by finite invariance and preserved across admissible collapse descriptions.

Scope and Limitations. This argument is intentionally minimal. It does not yet constitute a full derivation of the Born rule in the sense of a general measure theorem over arbitrary Hilbert spaces. In particular:

- the argument is developed here for finite sector decompositions, especially the two-sector case;
- the extension from rational to general weights depends on admissibility and parameterization consistency rather than a fully formal continuity theorem;
- the origin of sector weights w_k from deeper collapse dynamics remains to be fully characterized.

Nevertheless, the result identifies Born-like scaling as the natural invariant weighting family on collapse-stable sectors, and therefore as a structural consequence of admissible collapse classes rather than an externally imposed probability postulate.

7 Representative Collapse Operators

The formulation developed in Sections 4–6 emphasizes that Quantum Collapse Geometry (QCG) is not defined by a single collapse operator, but by invariant relational structure preserved across admissible collapse classes. Nevertheless, explicit constructions play an important role in illustrating how such structure arises in concrete systems.

In this section, we present representative collapse operators drawn from distinct subclasses of $\mathcal{C}$. These examples are not intended as fundamental laws, but as realizations of collapse dynamics that exhibit the invariant families identified above.

7.1 Discrete Phase Alignment Operator

A prototypical example arises in discrete relational systems defined on a graph $G = (V, E)$, where each vertex carries a phase variable $\theta_i \in S^1$.

Define the update rule

$$\Phi(\theta)_i = \arg\left(\sum_{j\sim i} e^{i\theta_j}\right).$$

This operator belongs to $\mathcal{C}_{\text{disc}}$ and satisfies:

- relational locality through dependence on neighboring sites,
- non-expansive discrepancy behavior (Section 6.1),
- attractor formation through iterative alignment (Section 6.2),
- preservation of topological sectors under continuous evolution (Section 6.4).

Under iteration, configurations converge toward locally coherent phase sectors while retaining global topological structure. This operator therefore realizes local and global invariant families simultaneously.

7.2 Nonlinear Measurement Operator

To model measurement-like behavior, consider a two-sector system with weights (w_0, w_1) satisfying $w_0 + w_1 = 1$. Define the update rule

$$\Phi(w)_i = \frac{w_i^\gamma}{w_0^\gamma + w_1^\gamma}, \qquad \gamma > 1.$$

This operator belongs to $\mathcal{C}_{\text{meas}}$ and exhibits:

- amplification of dominant components,
- convergence to fixed-point sectors $(1, 0)$ and $(0, 1)$,
- basin partitioning corresponding to measurement outcomes.

The invariant family realized here is the set of fixed-point sectors and their associated basins of attraction, providing a concrete realization of measurement as sector formation (Section 6.3).

7.3 Continuum Kernel Collapse Operator

To extend collapse dynamics to continuous systems, consider a phase field $\theta(x)$ defined over a domain $\Omega \subset \mathbb{R}^n$.

Define the operator

$$\Phi[\theta](x) = \arg\left(\int_\Omega K(x, x')e^{i\theta(x')}\,dx'\right),$$

where $K(x, x')$ is a localized kernel.

This operator belongs to $\mathcal{C}_{\text{cont}}$ and exhibits:

- quasi-local interaction via kernel support,
- smoothing behavior analogous to diffusion in the local limit,
- preservation of global phase structure under continuous evolution.

In the small-neighborhood limit, repeated application of Φ yields dynamics consistent with reduction of local phase variation, providing a continuum realization of the local alignment invariant family.

7.4 Attractor-Induced Ensemble Models

Ensemble behavior can be illustrated by considering repeated application of collapse dynamics to an ensemble of initial configurations.

Let $\Phi \in \mathcal{C}$ partition Σ into basins $\{B_k\}$. Define outcome frequencies by

$$f_k = \int_{B_k} \rho(x)\,d\Sigma,$$

for some admissible measure ρ.

This construction realizes the measure invariant family described in Section 6.5, where probability emerges as a weighting over attractor basins rather than as an intrinsic stochastic element of the dynamics.

7.5 Summary of Operator Roles

The examples above illustrate a key principle:

> Different collapse operators realize different aspects of invariant relational structure, but no single operator is privileged.

- Discrete alignment operators emphasize local consistency and topological preservation,
- nonlinear measurement operators emphasize sector formation,
- continuum operators emphasize smoothing and field-level behavior,
- ensemble constructions emphasize measure invariance and probability.

All such operators belong to subclasses of $\mathcal{C}$ and are unified by the invariant families they preserve.

7.6 Interpretation

The role of explicit collapse operators is therefore illustrative rather than foundational. They provide concrete realizations of the abstract structure defined by collapse classes, but the physically meaningful content of QCG resides in the invariant families identified across these constructions.

In this sense, collapse operators play a role analogous to coordinate systems in geometry: they provide a means of representation, but do not define the underlying structure.

8 Categorical Realization of Collapse Structure

8.1 Purpose and Positioning

The collapse-class framework developed in this work is formulated at a structural level, independent of any specific mathematical representation. Nevertheless, it is useful to exhibit a concrete realization of collapse-consistent structure within an established formal setting.

In this section, we present such a realization using the category **CPM(FHilb)**, the category of finite-dimensional Hilbert spaces and completely positive maps. This provides a mathematically well-defined environment in which the structural features of collapse—selection, decoherence, and invariant sector formation—can be represented explicitly.

It is important to emphasize that this categorical formulation is not intended as a foundational ontology. Rather, it should be understood as an effective representation of invariant relational structure under admissible collapse dynamics. The role of category theory here is descriptive, not generative.

8.2 The Category CPM(FHilb)

Let

$$\mathcal{Q} \equiv \mathbf{CPM}(\mathbf{FHilb})$$

denote the category whose objects are finite-dimensional Hilbert spaces and whose morphisms are completely positive (CP) maps.

This category is †-compact and monoidal, supporting composition, tensor products, and duals. It also admits classical structures represented by special commutative †-Frobenius algebras, which provide a natural representation of classical data within a quantum setting.

Within this framework:

- objects correspond to relational configuration spaces,
- morphisms correspond to admissible transformations,
- composition corresponds to sequential application of collapse dynamics.

8.3 Decoherence as a Selection Functor

We model the suppression of incompatible relational structure by an endofunctor

$$D : \mathcal{Q} \to \mathcal{Q}.$$

This functor satisfies:

- **Idempotence:** $D^2 \cong D$,
- **Structure preservation:** D is monoidal and respects composition.

Interpretationally, D implements finite invariance: it removes distinctions that cannot persist under admissible collapse dynamics and exposes coherence-stable structure.

8.4 Instruments and Per-Outcome Collapse

Let C_B be a classical object in $\mathcal{Q}$, represented by a special commutative †-Frobenius algebra.

A measurement-like process is represented by an instrument

$$\mathrm{Instr}_B : H \to H \otimes C_B.$$

For each outcome $b \in B$, define the corresponding branch map

$$\kappa_b = (\mathrm{id}_H \otimes \varepsilon_b) \circ \mathrm{Instr}_B : H \to H.$$

These maps represent the decomposition of collapse into outcome-specific branches.

8.5 Global Collapse as an Idempotent Operator

The global collapse operator is defined by

$$\mathrm{Coll}_H = \left(\sum_{b \in B} \kappa_b \right) \circ D_H.$$

This operator satisfies:

- **Idempotence:** $\mathrm{Coll}_H \circ \mathrm{Coll}_H \cong \mathrm{Coll}_H$,
- **Non-invertibility:** information is irreversibly reduced,
- **Sector projection:** collapse maps configurations into invariant sectors.

Thus collapse acts as a projector onto stable relational structure.

8.6 Naturality and Structural Consistency

Collapse must be compatible with admissible transformations. For any morphism $f : H \to K$, we require

$$\mathrm{Coll}_K \circ f \cong f \circ \mathrm{Coll}_H.$$

This naturality condition ensures that collapse preserves invariant relational structure across mappings.

8.7 Invariant Subobjects and Stable Sectors

Define the invariant sector

$$I_H = \mathrm{Im}(\mathrm{Coll}_H).$$

These subobjects correspond to collapse-stable configurations. They are the categorical realization of admissible sectors introduced earlier.

8.8 Classical Interface and Descriptive Projection

The classical object C_B provides a categorical realization of the observable space $\mathcal{O}$. Interaction with this object implements descriptive projection, replacing the abstract mapping $P : \Sigma \to \mathcal{O}$ with a concrete categorical construction.

8.9 Collapse as a Comonad-Compatible Algebraic Structure

To refine the categorical realization of collapse, we separate two distinct structural roles: the enforcement of admissibility through decoherence, and the selection of stable sectors through classical branch structure.

This separation admits a natural formulation in which decoherence is modeled by a comonad, while branch selection is expressed as an algebra over a classical endofunctor.

Base category. Let

$$\mathcal{Q} \equiv \mathbf{CPM}(\mathbf{FHilb})$$

denote the category of finite-dimensional Hilbert spaces and completely positive maps.

Decoherence as an idempotent comonad. Let

$$D : \mathcal{Q} \to \mathcal{Q}$$

be an idempotent comonad with counit

$$\varepsilon_H : DH \to H$$

and comultiplication

$$\delta_H : DH \to D^2H,$$

satisfying the comonad identities and

$$D^2 \cong D.$$

Interpretationally, D enforces finite invariance: it removes distinctions that cannot be maintained under admissible collapse dynamics and restricts attention to coherence-stable relational structure.

D-coalgebras and admissibility. A D-coalgebra is a pair (H, γ_H) with

$$\gamma_H : H \to DH$$

satisfying

$$\varepsilon_H \circ \gamma_H = \mathrm{id}_H, \qquad \delta_H \circ \gamma_H = D(\gamma_H) \circ \gamma_H.$$

These coalgebras encode systems equipped with admissibility structure. In QCG terms, they represent relational configurations together with their embedding into the finite-invariance regime.

Classical branch structure and induced monad. Let C_B be a classical object in $\mathcal{Q}$, represented by a special commutative †-Frobenius algebra. In particular, C_B carries a monoid structure

$$\mu_B : C_B \otimes C_B \to C_B, \qquad \eta_B : I \to C_B.$$

This induces an endofunctor

$$T(X) = X \otimes C_B$$

with monad structure

$$\eta_X = \mathrm{id}_X \otimes \eta_B, \qquad \mu_X = \mathrm{id}_X \otimes \mu_B.$$

Interpretationally, T represents classical branching structure: adjoining a classical register that encodes outcome labels.

Selection as a T-algebra. A T-algebra on the decohered system DH is a morphism

$$a_H : DH \otimes C_B \to DH$$

satisfying the algebra axioms:

$$a_H \circ (\mathrm{id}_{DH} \otimes \eta_B) = \mathrm{id}_{DH},$$

$$a_H \circ (a_H \otimes \mathrm{id}_{C_B}) \;\cong\; a_H \circ (\mathrm{id}_{DH} \otimes \mu_B).$$

This map implements classical control: it selects a stable relational configuration conditioned on a classical branch label.

Instrument and induced selection map. Let

$$\mathrm{Instr}_B : DH \to DH \otimes C_B$$

be an instrument defined on the decohered system.

We define the induced selection endomorphism

$$S_H := a_H \circ \mathrm{Instr}_B : DH \to DH.$$

Thus selection is not primitive, but arises from the interaction between branching structure and algebraic resolution.

Global collapse. The global collapse operator on H is defined by

$$\mathrm{Coll}_H = \varepsilon_H \circ S_H \circ \gamma_H = \varepsilon_H \circ a_H \circ \mathrm{Instr}_B \circ \gamma_H.$$

In the idempotent regime where H is identified with its decohered image, this reduces to

$$\mathrm{Coll}_H = a_H \circ \mathrm{Instr}_B.$$

This recovers the structural decomposition

$$\mathrm{Coll} = S \circ D,$$

now with explicit algebraic meaning.

Comonad compatibility. For collapse to preserve admissibility, it must be compatible with the D-coalgebra structure. We require

$$\gamma_H \circ \mathrm{Coll}_H \;\cong\; S_H \circ \gamma_H,$$

ensuring that collapse acts within the admissible regime defined by the comonad.

Idempotence and invariant sector. Because D is idempotent and a_H resolves stable branch structure, collapse is idempotent up to canonical isomorphism:

$$\mathrm{Coll}_H \circ \mathrm{Coll}_H \;\cong\; \mathrm{Coll}_H.$$

The image

$$I_H = \mathrm{Im}(\mathrm{Coll}_H)$$

defines the collapse-stable sector. This subobject inherits admissibility structure and corresponds to the invariant relational configuration identified in earlier sections.

Interpretation. This formulation makes explicit the layered structure of collapse:

1. the comonad D enforces finite invariance by restricting configurations to coherence-stable relational structure;
2. the monad $T(X) = X \otimes C_B$ encodes classical branching structure;
3. the algebra a_H selects stable sectors conditioned on branch labels;
4. the composite Coll_H acts as an idempotent projector onto invariant relational structure.

Thus collapse is not itself a primitive comonad, but a *comonad-compatible algebraic selection operator*. This preserves the ontological ordering of QCG: categorical structures provide a formal realization of collapse-consistent physics, but do not replace collapse as the generative primitive.

8.10 Relation to QCG Ontology

This categorical formulation realizes the core ontological elements of QCG:

- collapse corresponds to an idempotent natural transformation,
- finite invariance corresponds to the comonad D,
- invariant sectors correspond to subobjects,
- admissibility corresponds to stability under Coll.

Category theory therefore provides a formal representation of collapse-consistent structure, but does not replace collapse as the primitive generative element.

8.11 Cross-Domain Structural Correspondence

The structure identified here extends beyond quantum systems. Analogous patterns appear in:

- probability theory (conditioning and projection),
- optimization (projection onto feasible sets),

- learning (regularization and constraint enforcement),
- biological evolution (selection dynamics).

This suggests that collapse, understood as constraint-driven selection, is a general structural principle appearing across domains.

8.12 Summary

The categorical formulation presented here provides a concrete realization of collapse structure within a well-established mathematical framework. It demonstrates that the core features of QCG—selection, decoherence, and invariant sector formation—can be expressed in precise algebraic terms.

At the same time, it reinforces the central thesis of this work: that such mathematical structures are representations of deeper relational invariants, rather than their ultimate source.

9 Emergent Physical Structure

The preceding sections have identified invariant families arising across admissible collapse classes. We now show how familiar physical structures can be understood as emergent from these invariant relational features.

In this formulation, physical quantities are not introduced as fundamental primitives, but arise as stable descriptors of invariant structure under collapse dynamics.

9.1 Energy as Relational Tension

A natural invariant associated with relational configurations is a measure of local inconsistency or tension. For phase-like variables, a representative functional is

$$E[x] = \sum_{\langle i,j\rangle \in E} \left(1 - \cos(\theta_i - \theta_j)\right),$$

which vanishes for locally aligned configurations.

As shown in Section 6.1, collapse dynamics in $\mathcal{C}$ tend to reduce or constrain such discrepancy functionals. In this sense, energy can be interpreted as a measure of relational tension, with collapse acting to minimize or stabilize it.

This interpretation does not require a fundamental Hamiltonian; rather, energy emerges as an invariant descriptor of local relational structure under admissible collapse dynamics.

9.2 Force as Gradient of Invariant Structure

Given a relational tension functional $E[x]$, one may define a generalized force as the direction of steepest change with respect to relational degrees of freedom:

$$F_i = -\frac{\partial E}{\partial \theta_i}.$$

In discrete systems, this corresponds to local alignment tendencies, while in continuous systems it yields differential operators governing field evolution.

Under collapse dynamics, such gradients are not imposed externally, but arise naturally from the tendency of configurations to evolve toward lower-discrepancy states. Force can therefore be understood as an emergent descriptor of invariant relational adjustment.

9.3 Time as Synchronization of Collapse Processes

Time is not introduced as a fundamental parameter in QCG, but emerges from the ordering and synchronization of collapse events.

As configurations evolve under iteration of $\Phi \in \mathcal{C}$, relational structures tend to stabilize at different rates depending on local conditions. Regions of Σ that exhibit coherent progression of collapse can be associated with an effective temporal ordering.

In this sense, time corresponds to the propagation of coherent relational updates across configuration space. It is therefore an emergent parameter derived from invariant patterns of synchronization rather than a background coordinate.

9.4 Geometry as Stabilized Coupling Structure

The relational configuration space Σ includes coupling relationships between degrees of freedom, represented for example by graph connectivity or interaction kernels.

Under collapse dynamics, configurations that maintain consistent relational structure over extended regions become stabilized. The pattern of stable couplings induces an effective geometry.

In discrete systems, this corresponds to persistent graph structure; in continuous systems, it corresponds to effective metric or connectivity properties derived from the kernel $K(x, x')$.

Geometry thus emerges as an invariant family of coupling relations preserved under collapse, rather than as a predefined background.

9.5 Quantization from Topological Invariance

As shown in Section 6.4, collapse dynamics can preserve global topological invariants such as winding number. These invariants take discrete values and are robust under local updates.

In this framework, quantization arises naturally from the discreteness of topological classes. Physical quantities associated with such invariants (e.g., charge-like quantities) are therefore quantized not by imposition, but by the structure of invariant families under collapse dynamics.

9.6 Measurement and Probability Revisited

Measurement and probability emerge from the invariant structures identified in Sections 6.3 and 6.5.

- Measurement corresponds to convergence into fixed-point sectors under collapse, together with projection to observable descriptions.
- Probability corresponds to invariant weighting over attractor basins induced by admissible ensemble structure.

These features are not additional postulates, but consequences of invariant relational structure across collapse classes.

9.7 Summary

The structures commonly treated as fundamental in physics—energy, force, time, geometry, quantization, measurement, and probability—can all be understood as emergent from invariant families under admissible collapse dynamics.

> Physical law corresponds to stable relational structure preserved across collapse classes.

This perspective shifts the role of physical theory from specifying fundamental equations of motion to identifying invariant structures that persist under admissible transformations of relational configuration space.

10 Interpretation and Relation to Existing Frameworks

The formulation developed in this work is not intended as a direct replacement for established physical theories, but as a structural framework for understanding the origin of effective physical descriptions. In this section, we clarify the interpretive status of Quantum Collapse Geometry (QCG) and its relation to existing approaches.

10.1 QCG as a Structural Framework

QCG should be understood as a theory of invariant relational structure rather than as a proposal for a specific fundamental dynamical law.

The central claim is not that a particular collapse operator governs physical systems, but that:

> Physical law corresponds to invariant families of relational structure preserved across admissible collapse classes under finite invariance.

In this sense, QCG operates at a level analogous to that of symmetry principles or renormalization frameworks: it identifies constraints on admissible structure rather than specifying a unique microscopic equation of motion.

10.2 Relation to Quantum Mechanics

Quantum mechanics provides a highly successful descriptive framework for physical systems, formulated in terms of Hilbert spaces, operators, and probabilistic measurement outcomes.

Within QCG, this formalism is reinterpreted as an *effective descriptive layer* arising from invariant families under collapse dynamics:

- Hilbert space structure encodes relations between invariant sectors of Σ,
- eigenstates correspond to collapse-stable sectors,
- projection corresponds to coarse-grained identification of attractor basins,
- probabilities correspond to invariant weighting over these basins.

This mapping does not derive quantum mechanics in full generality, but provides a structural interpretation of its core elements within the collapse-class framework.

10.3 Comparison with Decoherence-Based Approaches

Decoherence-based programs explain the emergence of classical behavior through interaction with an environment, leading to suppression of interference and the selection of preferred states.

QCG shares with these approaches the emphasis on effective structure and emergent behavior. However, it differs in that:

- collapse is treated as a universal update grammar rather than an environmental effect,

- invariant sectors arise from internal relational dynamics rather than external coupling,
- probability is interpreted as a structural weighting over admissible configurations.

In this sense, decoherence can be viewed as a particular realization of effective collapse within a broader class of admissible dynamics.

10.4 Comparison with Deterministic Completion Theories

Deterministic completion theories, such as trajectory-based formulations, introduce additional ontological structure to recover definite outcomes.

QCG similarly provides definite sector selection through collapse dynamics, but differs in that:

- it does not introduce additional hidden variables beyond relational configuration,
- it does not require a unique guiding equation,
- it attributes probabilistic behavior to invariant structure rather than to ignorance over underlying states.

Thus, QCG may be viewed as a structural alternative to deterministic completion approaches, focused on invariant families rather than trajectory-level ontology.

10.5 Comparison with Objective Collapse Models

Objective collapse models modify quantum dynamics by introducing stochastic terms that produce state reduction.

QCG shares the goal of explaining definite outcomes without external measurement, but differs in key respects:

- collapse is not introduced as a stochastic modification of a prior linear theory,
- no specific dynamical law is privileged,
- collapse is interpreted as a class of admissible relational updates rather than as a physically specified noise process.

In this sense, objective collapse models may be viewed as particular instantiations within a broader space of collapse dynamics.

10.6 Relation to Effective Theory Frameworks

The role of QCG is most closely aligned with effective theory frameworks in modern physics, such as renormalization group approaches.

In these frameworks:

- fundamental behavior is not directly observable,
- effective laws emerge at different scales,
- invariant structure under transformation plays a central role.

Similarly, QCG identifies invariant relational structures that persist across admissible collapse dynamics, with familiar physical laws emerging as effective descriptions of these invariants.

10.7 What QCG Does and Does Not Claim

It is important to delineate the scope of the present work.

QCG does *not* claim:

- to provide a complete derivation of quantum mechanics from first principles,
- to specify a unique fundamental dynamical law,
- to make immediate experimental predictions beyond existing frameworks.

QCG does claim:

- that collapse can be formulated as a class of admissible relational dynamics,
- that invariant families across these dynamics provide the correct level of physical description,
- that key features of quantum and classical physics can be reinterpreted as emergent from such invariant structure.

10.8 Interpretive Summary

The perspective developed here suggests a shift in emphasis:

> Physical theories should not be understood solely as equations governing fundamental entities, but as descriptions of invariant relational structure under admissible transformations.

In this view, collapse dynamics define the space of possible evolutions, while invariant families define the physically meaningful content of the theory.

QCG thus provides a framework in which the distinction between fundamental law and emergent structure is reframed: what is fundamental is not a specific dynamical rule, but the invariant relational structure that persists across admissible collapse dynamics.

11 Limitations and Future Work

The formulation of Quantum Collapse Geometry (QCG) presented here establishes a structural framework based on invariant families across admissible collapse classes. While this framework unifies a range of conceptual and model-based results, it remains incomplete in several important respects. We summarize the principal limitations and outline directions for further development.

11.1 Lack of a Complete Classification of Collapse Classes

The notion of a collapse class $\mathcal{C}$ is defined here through a set of structural conditions (Section 4), but no complete classification of admissible collapse dynamics has been provided.

In particular:

- the minimal set of conditions required to recover known physical structure remains to be fully characterized,
- the relationships between different subclasses (e.g., discrete, continuous, topological) are not yet systematically understood,
- the extent to which physically relevant dynamics form a restricted subset of $\mathcal{C}$ is an open question.

Future work should aim to identify canonical subclasses and characterize their invariant families more precisely.

11.2 Incomplete Characterization of Invariant Measures

The emergence of probability as an invariant weighting over attractor basins (Section 6.5) relies on the existence of an admissible measure ρ over relational configurations.

However:

- the origin of ρ from underlying collapse dynamics is not uniquely determined,

- the structural constraints leading to Born-like scaling are not yet derived from first principles within the framework,
- extension to general multi-sector and continuous systems requires further analysis.

A central open problem is to determine whether the properties imposed on the weighting functional can be derived directly from the geometry of admissible sets under collapse.

11.3 Continuum Limit and Field-Theoretic Structure

While continuum collapse operators have been introduced (Section 7), the connection between these constructions and established field-theoretic frameworks remains incomplete.

In particular:

- the relationship between collapse dynamics and unitary evolution has not been formally established,
- the emergence of relativistic structure and causal constraints is not yet addressed,
- the role of gauge symmetry and higher-dimensional field structure remains open.

Developing a consistent continuum and field-theoretic formulation of collapse classes is essential for connecting QCG to known physical theories.

11.4 Topological Structure Beyond Minimal Models

The preservation of topological invariants has been demonstrated in representative systems (Section 6.4), but a general theory of topological invariant families under collapse dynamics is not yet available.

Key open questions include:

- extension to higher-dimensional and non-abelian topological structures,
- characterization of singularities and topological transitions under collapse,
- identification of invariant families corresponding to known gauge-theoretic quantities.

Further work is needed to determine the extent to which topological protection is a generic feature of admissible collapse classes.

11.5 Relation to Experimental Observables

At present, QCG is formulated as a structural framework and does not provide new quantitative predictions that distinguish it experimentally from existing theories.

Specifically:

- no unique collapse operator is specified, preventing direct parameter estimation,
- the framework reproduces known qualitative features without yet yielding new testable deviations,
- the connection between invariant families and measurable quantities requires further refinement.

An important direction for future work is the identification of regimes in which different collapse subclasses produce distinguishable effective behavior.

11.6 Mathematical Formalization

Many results in this work are presented at a structural or schematic level. While this is appropriate for establishing the framework, greater mathematical rigor is required.

In particular:

- formal definitions of collapse classes in functional-analytic terms,
- existence and uniqueness results for invariant families,
- rigorous treatment of attractor structure in infinite-dimensional spaces,
- extension of the weighting argument in Section 6.5 to general measure-theoretic settings.

Establishing a fully rigorous mathematical foundation for QCG remains a major task.

11.7 Programmatic Outlook

Despite these limitations, the framework developed here suggests a coherent research program:

- classify admissible collapse classes and their invariant families,
- derive structural constraints on invariant measures from collapse geometry,

- develop continuum and field-theoretic realizations of collapse dynamics,
- identify connections between invariant families and known physical symmetries,
- explore regimes in which collapse-class variation leads to observable effects.

11.8 Summary

QCG, as presented here, is best understood as a structural program rather than a completed theory. Its central contribution is to shift the focus of physical description from specific dynamical laws to invariant relational structure across admissible transformations.

> The task of the theory is not yet complete; it is to identify the invariant structure from which physical law emerges.

12 Conclusion

In this work, we have reformulated Quantum Collapse Geometry (QCG) from an operator-centric approach to a structural framework based on invariant relational properties. By introducing collapse classes and invariant families, we have shifted the focus from the specification of particular collapse dynamics to the identification of structures that persist across admissible transformations of relational configuration space.

Within this formulation, key features of physical description arise naturally from invariant structure. Local alignment and discrepancy reduction characterize the suppression of relational inconsistency; attractor structure defines admissible configurations and sector formation; measurement emerges as the identification of invariant sectors under projection; topological invariants persist as global constraints; and probability appears as an invariant weighting over attractor basins under admissible ensemble structure.

These results collectively support a reframing of physical theory. Rather than seeking a privileged dynamical rule governing fundamental entities, the present framework suggests that the physically meaningful content of a theory resides in the structures that remain stable across admissible dynamics.

> Physics arises not from a privileged collapse operator, but from invariant relational structures preserved across admissible collapse dynamics under finite invariance.

This perspective aligns with broader developments in modern physics, in which symmetry, invariance, and effective description play central roles. At the same time, it extends these ideas by identifying collapse—understood as a universal update grammar—as the mechanism through which invariant relational structure is selected and stabilized.

The framework presented here is not a complete theory, but a program. Its central task is to classify admissible collapse classes, characterize their invariant families, and determine how known physical laws emerge as effective descriptions of these structures.

If successful, this approach suggests that the search for fundamental law may be reframed: not as the discovery of a unique equation of motion, but as the identification of the invariant relational structures that persist across all admissible forms of dynamical evolution.